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1 Gauss's Law

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Formally, we say that the electric flux around an enclosed surface gives us the enclosed charge. The epsilon is really the permittivity of free space which may change if we put the charged particle in another medium.

Choose symmetric surface.

Rule of Thumb: conducting materials only have charge on the surface since they same charges tend to repel one another.

1.1 Important Examples:

1.1.1 Sphere:

When inside the sphere, $r < r_0$:

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$Q_{encl} = \frac{\rho_E V_{outer}}{\rho_E V_{inner}} Q_{total}$$

$$Q_{encl} = \frac{r^3}{r_0^3} Q_{total}$$

$$E \cdot 4\pi r^2 = \frac{Q_{total}}{\epsilon_0} \cdot \frac{r^3}{r_0^3}$$

$$E = \frac{Q_{total}}{4\pi\epsilon_0} \cdot \frac{r}{r_0^3}$$

When outside the sphere, $r \geq r_0$:

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

$$E \cdot 4\pi r_0^2 = \frac{Q_{total}}{\epsilon_0}$$

$$E = \frac{Q_{total}}{4\pi r_0^2 \epsilon_0}$$

1.1.2 Line:

Assume line is infinitely long. Our Gaussian surface is a cylinder. Sides have $\vec{E} \perp d\vec{A}$ so they are zero. Charge per unit length is λ

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Sides are 0

$$E \cdot 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi r \epsilon_0}$$

1.1.3 Surface:

Assume the surface is infinitely long. (Don't care if it is conductive) Charge per area is σ. Use a cylinder as gaussian surface all the way through the plane.

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Side is 0. Top and bottom are left.

$$E \cdot A + E \cdot A = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

1.1.4 Capacitors:

Similar to surface except we use a conductive material and draw the cylindrical only for the surface layer.

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Side is 0, bottom is zero because no electric field inside the conductor.

$$E \cdot A = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$

2 Electric Potential

Rule of thumbs

1. Electric field is analogous to gravitational field.
2. Electric field points towards (-).
3. Positive charges follow the electric field. Negative charges go in the reverse.
4. Electric field and equipotential lines are perpendicular.
5. Electric field points towards lower Voltage values.
6. Two of the same charge creates equipotential lines like boobies and an electric field who avoids each other.
7. Two different charges creates equipontential lines that are meshed against each other and an electric field that is like a sink and a source.
8. Always goes in direction of lower potential energy.

$$\vec{F} = q\vec{E}$$

$$U_a = q \cdot V_a$$

Think of U as the absolute potential energy while V is the electric potential energy per unit charge. So basically U includes the amount of coulombs in its measurements while V excludes the coulombs and measures the measures the amount of electric potential energy per couloumb.

But remember that only differences in electric potential is measurable. You can't really "measure" absolute gravitation potential energy. You always have to measure it relative to something like the ground. That makes the measurements meaningful.

Since voltage is **always** the difference in electric potential energy anyways it is extremely common to not write the delta.

$$\Delta U = q \cdot V_{ab} = q \cdot \Delta V$$

$$W = -\Delta\vec{U} = q\Delta\vec{V}$$

Once again, the analogous equations:

$$\Delta U = -\int_C \vec{F} \cdot \vec{dl}$$

$$V_{ab} = \Delta V = -\int_C \vec{E} \cdot \vec{dl}$$

Note: we use dl when talking about a 2d path integral.

Voltage here is the voltage

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Potential energy due to two point charges (analogous to Newton's Gravitational Force):

$$\Delta U = qV_{ab} = \frac{1}{4\pi\epsilon_0}\frac{q \cdot Q}{r}$$

2.0.1 E and V

We can expand upon the relationship between Electric field and voltage. We've been mostly dealing with 2d path integrals along a 2d shape. However, we can expand upon this. Basically the dot product between electric field and our direction in 3d space gives us the incremental voltage change.

$$dV = -\vec{E} \cdot \vec{dr}$$

In integral form:

$$V = -\int \vec{E} \cdot \vec{dr}$$

Which is the same as:

$$\vec{E} = -\vec{\nabla} V$$

Which tells us that the highest gradient descent for the Voltage function $V(x, y, z)$ is the path for the electric field. This makes intuitive sense as they are perpendicular at every point and the electric field points towards lower voltage values.

Note that electric field is vector valued and has components like forces. Hence, for $V(x, y)$, when we give the partial derivatives, we get the electric field's vector components.

2.0.2 Electron Volt

Do not be fooled, the electron volt does not measure voltage. Joules are too large for electrons and atoms. $1eV = 1.6022 \times 10^{-19} J$ This is the energy required to move one electron across one volt.

$$W = qV = e \times 1V = eV$$

where e is the charge of an electron.

2.1 Particle Accelerator Cathode Ray Tubes (Very basic)

Literally just a capacitor

$$q \cdot V_{ab} = \Delta U$$

Conservation of energy. Here all the potential energy gets converted into kinetic energy.

$$\Delta U + \Delta K = 0$$

$$-q \cdot V_{ab} = \frac{1}{2}mv^2$$

$$\sqrt{\frac{-2q \cdot V_{ab}}{m}} = v$$

So with so with that much voltage, we can only accerate to a certain speed. Notice that the speed depends on m and that if the charge is positive our calculations wouldn't make any sense.

2.2 Capacitors

Imagine a negative charge going from the negative plate to the positive plate. Remember that E points towards the (-):

$$V_{ba} = -\int_a^b \vec{E} \cdot \vec{dl}$$

An electron travels opposite to the electric field (from negative plate to positive plate). So $\vec{E} \cdot \vec{dl} = -E \cdot dl$

$$V_{ba} = E \int_a^b \vec{dl}$$

$$V_{ba} = E \cdot (b - a)$$

2.3 Sphere

For a **conducting** sphere of some charge. For example, a Tesla coil.

$$E = \frac{Q}{4\pi\epsilon r^2}$$

$$V_{ba} = -\int_{r_a}^{r_b} \frac{Q}{4\pi\epsilon_0 r^2} dl$$

$$V_{ba} = -\int_{r_a}^{r_b} \frac{Q}{4\pi\epsilon_0 r^2} dl$$

$$V_{ba} = \frac{Q}{4\pi\epsilon_0}(\frac{1}{r_b} - \frac{1}{r_a})$$

We get the voltage measured from comparing a sphere's charge at different radiuses.

Outside the sphere ($r > r_0$):
Let's set that $r_a = \infty$ so we can measure relative to $V_a = 0$.

$$V_{ba} = \frac{Q}{4\pi\epsilon_0 r}$$

Inside the sphere ($r \leq r_0$):

$$V_{ba} = \frac{Q}{4\pi\epsilon_0 r_0}$$

Note that inside the sphere, voltage is a constant. Outside the sphere, it drops off at $1/r$.

Why is it not 0? Yes, E is zero inside the conducting sphere. That would mean the Voltage is zero only if you travel from point a to point b inside the sphere. In other words compared to each other, those two points are equipotential. So the inside of the sphere is equipotential. Then it must be equipotential with the surface. This makes sense because any conducting metal with a certain charge on it will be equipotential.

Note that outside the sphere $E = \frac{Q}{4\pi\epsilon_0 r^2}$ and inside the sphere $E = 0$.

$$V \propto \frac{1}{r} \textbf{ and } E \propto \frac{1}{r^2}$$

Note that the curve is flipped across the x axis for negative charges.

2.4 Breakdown Voltage, Lightning, Tesla Coils, Spark

When voltage is high enough, air can become ionized such that free electrons in the air is accelerated fast enough to knock electrons out of O_2 and N_2 molecules. This causes the air to become "conductive"

At 1 atm, breakdown strenght of air is $E_{BD} = 3 \times 10^6 V/m$

$$\frac{kQ}{r_0} = r_0 \times \frac{kQ}{r_0^2}$$
$$V_{BD} = r_0 \cdot E_{BD}$$

This shows that for sharp points, like tips of wires or lightning rods, the V_{BD} is lower due to smaller radius. High voltages can create a glow called **corona discharge**. Electrons are thrown off and when recombined with their molecules, emitt light. This breakdown voltage is measured relative to the ground.

2.5 Voltages are Level Curves!

Voltage is not like forces in the sense that they can be broken into "components." It is important to imagine voltages from point charges on a 2d plane as level curves or 3d surfaces. Now realistically, when we talk about point charges in 3d dimensions, it's harder to imagine, so keep coming back to this analogy.

Hence, to find the voltage at a point in space from 2 point charges, you measure the distances to each point charge and solve.

$$V = V_{Q_1} + V_{Q_2}$$

$$V = k\frac{Q_1}{r_1} + k\frac{Q_2}{r_2}$$

2.6 Hemholtz Coils and Millikan's Oil Drop Experiment

Let's find the voltage at point that is x distance away from the center of a uniformly charged ring of total charge Q (perpendicular to the area). $r = \sqrt{R^2 + x^2}$ where R is the radius of the coil.

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2 + x^2}}$$

Let's find the voltage at point that is x distance away from the center of a uniformly charged disk.

Because it is uniformly charged, 1/4 of the surface area must mean 1/4 of the total charge.

$$dA = 2\pi R dR$$

$$\frac{dq}{Q} = \frac{dA}{A} = \frac{2\pi R dR}{\pi R_0^2}$$

$$V = k\int \frac{dq}{d} = k\int Q\frac{2\pi r \cdot dr}{\pi R_0^2} \cdot \frac{1}{\sqrt{r^2 + x^2}}$$

$$= \frac{2kQ}{R_0^2}\int_0^{R_0} \frac{r \cdot dr}{\sqrt{r^2 + x^2}}$$

$$= \frac{kQ}{R_0^2}[2\sqrt{r^2 + x^2}]_0^{R_0}$$

$$= \frac{Q}{2\pi\epsilon_0 R_0^2}[\sqrt{R_0^2 + x^2} - x]$$

$$= \frac{Qx}{2\pi\epsilon_0 R_0^2}[(\frac{R_0^2}{x^2} + 1)^{1/2} - 1]$$

2.7 Electric Dipole

NOT magnetic dipole. At point P, there are two point charges of opposite charges connected by a rod of length l. This is analogous to a polar molecule.

$$V = k\frac{Q}{r} + k\frac{-Q}{r + \Delta r}$$

$$V = kQ(\frac{1}{r} - \frac{1}{r + \Delta r}) = kQ(\frac{\Delta r}{r(r + \Delta r)})$$

When $r \gg l$ then $\Delta r \simeq lcos(\theta)$ (if it's too close, you can't form a right triangle):

$$= \frac{1}{4\pi\epsilon_0}\frac{Qlcos(\theta)}{r^2}$$

$$p = Ql$$

Our expression shows us the voltage when the angle can make items cancel out. When $\theta \simeq \frac{\pi}{2}$ then we almost get no voltage due to cancelation. When $\theta \simeq 0$, the Q becomes Ql.

p is the dipole moment. This is very different from the moment of inertia.

3 Capacitors

Note that this relationship works at all moments in time. However it is most useful at equilibrium for a DC circuit. Two such scenarios are when charge is trapped on a plate (floating nodes) or the circuit has charged up the capacitor until its equilibrium.

If you attach a capacitor to a DC source, it technically immediately gains Q on one plate because there was already Q in the battery terminal. If you add an resistor, voltage across the capacitor is going to increase over time as the current slows down. The voltage across the resistor is going to adjust accordingly. Resistor is what causes the charging to be logarithmic. For simple DC circuits, its often easier (don't have to use calculus) to just look at the moments at equilibrium.

From experimental relationships:

$$Q = CV$$

$$dQ = C \cdot dV$$

Derived from our Electric field and voltage relationship in the previous chapter:

$$E = Vd$$

Because $E = \sigma/\epsilon_0$ and $\sigma = Q/A$:

$$E = \frac{Q}{\epsilon_0 A}$$

Found experimentally what contributes towards capacitance.

$$C = \epsilon_0 \frac{A}{d}$$

Parallel and Series (which expand to more capacitors):

$$C_{series} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{parallel} = C_1 + C_2$$

Potential energy and work in capacitors.

We can compare dielectrics relative to the free permittivity of space. Capacitors with higher dielectrics can store more charge. ϵ basically measures "resistance" towards electrons jumping across.

$$C = k \cdot C_0$$

$$\epsilon = k \cdot \epsilon_0$$

Condensor microphones are sensitive capacitors where flimsy "plates" move and change the capacitance, which changes the total voltage. Moving coil microphones rely on induction which is discussed later.

1. $V = \frac{Q_0}{CK}$
2. $V = \frac{V_0}{K}$
3. $E = \frac{E_0}{K}$
4. $E_0 - E_{ind} = E = \frac{E_0}{K}$
5. $E_{ind} = E_0(1 - 1/K)$

3.1 Odd-Shaped Capacitors

3.1.1 Cylindrical

Let's say we have a coaxial cylindrical shell around the inner wire with radiuses R_a and R_b respectively.

We can try using the capacitor definitions but it yields an unsolvable integral:

$$C = \epsilon_0 \frac{A}{d} = \int \epsilon_0 \frac{2\pi r l}{dr} = 2\pi\epsilon_0 l \int \frac{r}{dr}$$

We can leverage the $Q = CV$ relationship instead:
Using Gauss's law $E = \frac{\lambda}{2\pi R \epsilon_0} = \frac{Q}{2\pi R l \epsilon_0}$ for a wire.
Then the voltage is:

$$V = -\frac{Q}{2\pi\epsilon_0 l} \int \frac{1}{r} dr = \frac{Q}{2\pi\epsilon_0 l} \ln \frac{R_a}{R_b}$$

Notice that the outer shell does not effect the electric field as we measure electric field relative to the enclosed charge. Hence, this means the external shell doesn't affect voltage.

$$C = \frac{2\pi\epsilon_0 l}{\ln R_a/R_b}$$

3.1.2 Spherical

Again the capacitance relationship fails us. So we'll follow the same procedure as above.

$$Q = CV$$

$$E = \frac{Q}{4\pi r^2\epsilon_0}$$

$$V = -\int \frac{Q}{4\pi\epsilon_0} dr = -\frac{Q}{4\pi r^2\epsilon_0}\int \frac{1}{r^2} dr = \frac{Q}{4\pi r^2\epsilon_0}(\frac{1}{r_b} - \frac{1}{r_a})$$

$$C = \frac{4\pi\epsilon_0}{\frac{1}{r_b} - \frac{1}{r_a}} = \frac{4\pi\epsilon_0 r_b r_a}{r_a - r_b}$$

3.1.3 Two wires

The electric field will superimpose on one another like the gravitational field. Why is this different from the examples above where we didn't consider electric field's superposition with on another. Note, enclosed charges work differently. Remember that $E = 0$ for inside conductive materials and the electric field is measured by what charges it encloses. In this example, no enclosed surfaces, instead we'll have superposition.

$$E = \frac{\lambda}{2\pi\epsilon_0 x} + \frac{\lambda}{2\pi\epsilon_0(d-x)}$$

Note that $Eanddl$ point in opposite directions.

$$V = \frac{\lambda}{2\pi\epsilon_0}\int_R^{d-R} \frac{1}{x} + \frac{1}{d-x} dx$$

$$V = \frac{\lambda}{2\pi\epsilon_0}[\ln x - \ln(d-x)]_R^{d-R}$$

$$V = \frac{\lambda}{\pi\epsilon_0}[\ln(d-R) - \ln R]$$

Since $d \gg R$

$$V \simeq \frac{Q}{\pi l\epsilon_0}\ln(d/R)$$

$$\frac{C}{l} \simeq \frac{\pi\epsilon_0}{\ln(d/R)}$$

3.2 Potential Energy

Often times, it is wayy easier to just look at the moments at equilibrium and compare the potential energies. Otherwise calculus is involved.

$$dW = Vdq$$

This is work done starting with $Q = 0$ or $U = 0$:

$$W = \int_0^Q q/Cdq = \frac{Q^2}{2C}$$

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

3.2.1 Pulling Apart Plates

If you increase the plate separation by two of a capacitor with Q and -Q on the plates (floating nodes), what happens to the potential energies. C then halves to then $U = Q^2/2(C/2$ which changes by $1/(1/2) = 2$

Intuitively, two opposite charged plates will attract one another. Pulling them apart increases the potential energies.

3.2.2 Pulling Apart Plates 2

A capacitor at equilibrium at x separation is connected to battery with voltage V. The plates are moved apart until 3x while being connected to the battery.

$$U_i = \frac{\epsilon_0 AV^2}{2x}$$

$$U_f = \frac{\epsilon_0 AV^2}{6x}$$

$$\Delta U_{cap} = -\frac{\epsilon_0 AV^2}{3x}$$

The potential energy decreases since capacitance is decreasing and the amount of charge and energy it can store decreases.

It may seem that work is just $-\Delta U_{cap}$. However, that is the work done by the capacitor and not the total work done by the system. The total work would be:

$$W_{on} = \int \vec{F} \cdot \vec{dl} = \int_x^{3x} QEdl$$

$$Q = \frac{\epsilon_0 A}{l} V \textbf{ and } E = \frac{V}{2l}$$

It may seem that the electric field should be $E = \frac{V}{l}$ but that is the total electric field caused by the other plate **and** itself. Each plate creates half the electric field. We want the electric field caused by the other plate only.

$$W_{on} = \frac{\epsilon_0 AV^2}{2} \int_x^{3x} \frac{1}{l^2} dl = -\frac{\epsilon_0 AV^2}{2} \frac{-2}{3x} = \frac{\epsilon_0 AV^2}{3x}$$

Work is positive since force applied for a distance in the same directions.

Due to conservation of energy. $W_{onsystem} = \Delta U_{cap} + \Delta U_{batt}$

$$U_{batt} = \frac{2\epsilon_0 AV^2}{3x}$$

3.3 Dielectrics

$$C = kC_0$$

$$\epsilon = k \cdot \epsilon_0$$

3.3.1 Inserting dielectric

You have an air filled capacitor connected to a constant voltage source V_0. Capacitance is C_0. You insert dielectric with K. Find Q.

$$Q_0 = C_0 V_0$$

In order for V to stay constant and C to change:

$$Q = CV_0$$

$$kQ_0 = kC_0 V_0$$

You have an air-filled battery except you disconnect it from the voltage source once it reaches V_0. With capacitance C_0 and an dielectric you now insert, what is the voltage.

For the charge to remain constant:

$$Q_0 = C_0 V_0$$

$$Q_0 = kC_0 \frac{1}{k} V_0$$

Since $V = \frac{V_0}{k}$, then $\frac{E_0}{k} = q\frac{V_0}{k}$. In this scenario, $E = \frac{E_0}{k}$.

3.3.2 Removing Dielectrics

A capacitor with dielectric $K = 3.4$ is connected to a V - volt battery. After the capacitor is fully charged, the battery is disconnected. With A and d. With the battery disconnected (which means charge is preserved, otherwise voltage potential would be conserved), you remove the dielectric.

Before $(V = V_i)$:

$$C_i = k\epsilon_0 \frac{A}{d} \text{ , } Q_i = C_i V_i \text{ , } E = V_i/d \text{ , } U = \frac{1}{2} C_i V_i^2$$

After $(Q_i = Q_f)$:

$$C_f = C_i/k \text{ , } Q_f = C_f V_f = \frac{C_i}{k} V_i k \text{ , } E = \frac{E_i k}{d} \text{ , } U = \frac{1}{2} \frac{C_i}{k} (V_i k)^2$$

3.3.3 Microscopic View of Dielectrics

Dielectric molecules will become oriented with the external electric field. If they are polar, positive ends will be attracted towards the negative plate and vice versa. If not, electrons will move towards the positive plate. The result is an induced electric field inside of the dielectric that opposes the external electric field. Because of the additive nature of electric field, the resulting net electric field is that some of the external field goes towards the charges on the surface of the dielectric instead of the opposite plate.

$$E_{net} = E_0 - E_{ind} = \frac{E_0}{K} \to E_{ind} = E_0(1 - \frac{1}{K})$$

$$\sigma_{ind} = \sigma_0(1 - \frac{1}{K})$$

$$Q_{ind} = Q_0(1 - \frac{1}{K})$$

Say you have a capacitor with A, d, K that is charged to V_0 and disconnected with Q on the plates.

$$C_0 = \epsilon_0 \frac{A}{d}$$

$$Q_0 = C_0 V_0$$

???

4 Current and Resistance

4.1 Battery

At the basic level, batteries are often redox reactions that move electrons.

4.1.1 Galvanic Cells

The anode (Cd) dissolves electrons while the cathode withdraws electrons.

$$Cd(s) + Cu^{2+}(aq) \to Cd^{2+}(s) + Cu(aq)$$

4.1.2 Some Rule of Thumbs

$$I = \frac{dQ}{dt}$$

$$V = IR$$

4.1.3 Resistors

$$R = \rho \frac{L}{A}$$

ρ is the resistivity which is a measure of resistivity inherent to the material at hand. We see that the units are in $\Omega \cdot m$ which makes sense.

$$\sigma = \frac{1}{\rho}$$

σ is known as the conductivity.

Temperature dependence resistors, which are used sometimes as a "sensor" in electronics. Note that this is analogous to the linear thermal expansion.

$$\rho_T = \rho_0[1 + \alpha \Delta T]$$

These work at high temperatures. Thermistors work at small and respond very quickly to temperature changes.

4.1.4 Power

$$P = \frac{dU}{dt} = \frac{d(qV)}{dt} = IV = \frac{V^2}{R} = I^2 R$$

Electric energy is transformed into thermal energy and light when the collisions of electrons move through a wire causes the net kinetic energy of the wire's atoms to increase. Power is measured in Watts, which is J/s. This is sort of a measure of how energy intensive something is. 5 Joules in 10 years is not the same as 5 Joules in 1 second. Don't be fooled but kWh is in just Joules.

4.1.5 AC

$$V = V_0 sin(\omega t)$$

$$I = I_0 sin(\omega t)$$

Assuming R is constant. V_0 is the peak voltage (amplitude). I_0 is the peak current (amplitude).

$$P = R I_0^2 sin^2(\omega t)$$

The average value of $sin^2(\omega t)$ is 1/2 which we'd get $\overline{I^2} = \frac{1}{2} I_0^2$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

4.2 Microscopic View

4.2.1 Definitions

When a potential difference is applied to ends of a wire, an electric field is created inside the wire parallel to the walls. The wire is a conductor but the electric field inside is not 0. First of all we create an electric field inside of the conductor. If charges were at rest, they'd find equilibrium at the surface of the wire and the electric field inside would be 0 (electrostatics). Here, charges are allowed to move under the electric field.

Current density $\vec{j}$ is the electric current per unit cross-sectional area. If you were to slice the wire and look at the cross section. You divide the cross section into many many pieces and look at the vector quantity of the charge going through that small area. You take the dot product that that with the vector $d\vec{A}$ over the entire cross-sectional area, we get the current for that slice. Note that $\vec{j}$ can exist for any point in space but later, we often use $\vec{j}(r,t)$ where r is the position along the wire and t is the time.

$$j = \frac{I}{A}$$

$$I = \int \vec{j} \cdot d\vec{A}$$

The direction of a positive charge is generally in the same direction as E. That is $\vec{j} \cdot \vec{E}$ is generally positive.

At a microscopic level, the movement of electrons are mostly "brownian" and they move in a zig-zag chaotic way. When an electric field is introduced, they have a tendency to accelerate in a direction. They are still chaotic, but the net movements are in a certain direction. They reach a steady state average speed when the force from the resistance in the wire causes the "acceleration" to become zero (think of it as friction). They reach a **drift speed** v_d.

4.2.2 Building

Where e are the electrons, delta Q is. n is the number of free electrons. $-e$ are the charges of the electrons. $-ne$ is the net charge that is in a given volume given a certain drift speed.

$$\Delta Q = n(Vol)(-e) = -n(Av_d\Delta t)e$$

$$I = -neAv_d$$

$$j = -nev_d$$

The positive j is opposite of the direction of the electrons.

$$j(r,t) = \rho(r,t)v_d(r,t)$$

I'm not sure if there is an intuition behind this. Rather this is derived from our definitions.

4.2.3 E inside the wire

Using the macroscopic approximation equations from before.

$$R = \rho\frac{l}{A} \text{ , } I = jA \text{ , } V = El$$
$$V = IR$$
$$El = (jA)(\rho\frac{l}{A})$$
$$El = jpl$$
$$E = jp$$
$$\vec{j} = \sigma\vec{E}$$

This is the microscopic analog of Ohm's law.

4.2.4 Continuity Equation

Microscopic analog of...

$$\nabla \cdot \vec{j} = -\frac{\partial \rho}{\partial t}$$

4.3 Superconductivity

At super low temperatures, resistivity of certain metals drop almost instantly to zero. Such materials are **superconducting**.

Connection of this with magnetism use cases in industry is confusing.

4.3.1 London Equations and the Meissner Effect

4.4 Neurons

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5 DC Circuits

We often need some device that drives electrical energy. (Battery, generator, light, sound). The potential difference between the terminals when no current flows is EMF (or electromotive force) ε. Note it is a misnomer and is not a force.

More generally emf is just potential difference without current flowing.

When charge tries to move across a battery, there exists hindrance against the free flow. Such resistance is internal resistance r.

1. EMF of battery is unconnected

2. terminal voltage is connected

Terminal voltage is the voltage when accounting for internal resistance. $V_{ab} = \varepsilon - Ir$. When no current exists $V_{ab} = \varepsilon$. So when you measure a battery with a multimeter, you are measuring the emf. When you measure the voltage across a battery in a dc circuit, it will always be less than the emf.

$$R_{series} = R_1 + R_2$$
$$R_{parallel} = \frac{R_1 R_2}{R_1 + R_2}$$

Red on red, black on black. Why? You want batteries in parallel like a remote controller would. That way the current draws on all the batteries (less on the weaker ones and more on the stronger ones). If not you create a short. Imagine you linked two AA batteries in a loop.

5.1 Kirchoff's Rules

5.1.1 Conservation of Charge

What goes in must go out.

5.1.2 Converation of Energy

In any cycles, what goes up must come down.

5.2 RC Circuits

5.2.1 Charging a capacitor

Imagine you have a battery, resistor, switch and a capacitor in a loop. We use emf as the battery as a representation of an "ideal" battery that gives out constant potential difference. In reality, we would have to factor in internal resistance.

$$\varepsilon = V_{resistor} + V_{cap}$$
$$\varepsilon = IR + \frac{Q}{C}$$
$$C\varepsilon = RC\frac{dQ}{dt} + Q$$
$$\frac{dt}{RC} = \frac{dQ}{C\varepsilon - Q}$$
$$\frac{\tau}{RC} = -\ln\left(C\varepsilon - Q\right)\Big|_0^Q$$
$$-\frac{\tau}{RC} = \ln\left(C\varepsilon - Q\right)/C\varepsilon$$
$$-\frac{\tau}{RC} = \ln 1 - \frac{Q}{C\varepsilon}$$

$$e^{-\frac{\tau}{RC}} = 1 - \frac{Q}{C\varepsilon}$$
$$Q = Q_0(1 - e^{-\tau/RC})$$
$$V = V_0(1 - e^{-\tau/RC})$$
$$I = \frac{\varepsilon}{R}e^{-\tau/RC}$$

RC is the time constant. It represents the amount of time to reach $(1 - e^{-1}) = 63\%$ of the full charge. It would theoretically take infinite time to reach the full charge.

5.2.2 Discharging a capacitor

Imagine you have a resistor, switch and a charged capacitor in a loop. When the switch is closed, it begins to discharge.

$$V_R = V_C$$
$$IR = \frac{Q}{C}$$
$$-\frac{dQ}{dt}R = \frac{Q}{C}$$
$$\frac{dQ}{Q} = -\frac{dt}{RC}$$
$$\ln\frac{Q}{Q_0} = -\frac{1}{RC}\tau$$
$$Q = Q_0e^{-\tau/RC}$$
$$V = V_0e^{-\tau/RC}$$
$$I = I_0e^{-\tau/RC}$$

This is exponential decay. Again RC is the time constant and represents time it takes to reach discharge .37 of the original value.

5.2.3 Time constant

The amount of time it takes to reach 0.5 of the voltage or current is given by: $0.5 = e^{-\tau/RC}$

The time factor in RC circuits make them incredibly useful. They can create flashing units, blinking units, electronic pacemakers.

Once it reaches a certain voltage, it discharges via an op-amp or diode, and resets.

5.3 Hazards

Large electric energy contains a lot of energy that must to released somehow.

1. heating tissue
2. stimulating nerves and muscles

1. 10mA can cause severe contraction immobilizing the ability to remove from hazard
2. 80mA - 100mA passes though torso for more than a second or two will cause heart to contract irregularly. **ventricular fibrillation**
3. 1A can cause burns depending on surface area.

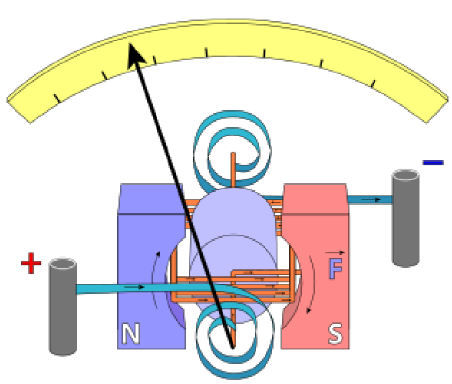
Current harms. Voltage drives the current. Voltage is like the potential difference between the top of a hill and the bottom, while current is the avalanche that is coming down due to that potential difference. The passing of these electrons at a fast rate contains capacity to harm.

Wet skin can cause resistance to be 1000Ω or less. If you are in good contact with the ground, $120V/1000\Omega = 120mA$ or $240V/1000\Omega = 240mA$ will definitely kill. You need to minimize current passing through you.

1. Thick insulating shoes
2. Dry hands
3. Don't complete a "circuit" through your heart. Keep your other hand in thy pocket
4. Electronic device grounds the outer shell.
5. Electronic device contains a grounding prong. While the "hot" wire already goes to the "neutral" wire. A ground wire makes sure that if anything goes really wrong internally the 120V is handled correctly.

5.4 Galvanometers

Basically, a



5.4.1 Ammeter

5.4.2 Voltmeter

6 Magnetism

An positive charge can exists independently from a negative charge. But a north pole cannot exist without a south pole. The absence of magnetic monopoles is observed in Maxwell's equation: $\oint \vec{B} \cdot d\vec{A} = 0$

1. Rule of thumb: an electric current produces a magnetic field.
2. RHR: Thumb points in direction of current. The fingers coil in direction of the magnetic field.
3. RHR: Orient your right hand like you're shooting a gun. You point in the direction of current (conventional???) Thumb is the force. Fingers are the
4. Rule of thumb: a magnet exerts a force on a current carrying wire.
5. Same current going in opposite directions in the same B field will have cancelling forces.

$$d\vec{F} = Id\vec{l} \times \vec{B}$$
$$d\vec{F} = q\vec{v} \times \vec{B}$$

Lorentz Equation. Force of both electric field and moving in a magnetic field.

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

6.1 Force on a semicircular wire

Imagine having a wire that traces a semicircle of radius R in an uniform magnetic field B going into the page. We know that any vector on the paper's plane will have the property $F = qvB$.

$$d\vec{F} = IBRd\theta$$

Due to symmetry, the x forces will cancel out.

$$F_y = \int_0^{\pi} dF sin\theta = \int_0^{\pi} IBRsin\theta d\theta = -IBRcos\theta|_0^{\pi} = 2IBR$$

6.2 Cyclotron

In an uniform constant magnetic field (hemholtz coils etc). Again uniform magnetic field going in the z axis means a circlular path on the xy plane represented by $F = qvB$.

$$F = qvB = ma_{cp}$$

* Remember that the centripetal acceleration is an acceleration where speed is constant but direction is always changing. Centripetal force does not exist in the real world, but rather is the replaced by a real world force like tension on a string, friction, normal force, or gravitational force etc.

$$qvB = m\frac{v_t^2}{r}$$

$$r = \frac{mv}{qB}$$

$$\tau = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

$$f = \frac{1}{\tau} = \frac{qB}{2\pi m}$$

6.3 Caveats

1. What is interesting is that a magnetic field has no power of stopping a charged particle. The force created is always perpendicular to its velocity, so a magnetic field can't generate an opposing force to decelerate the particle.

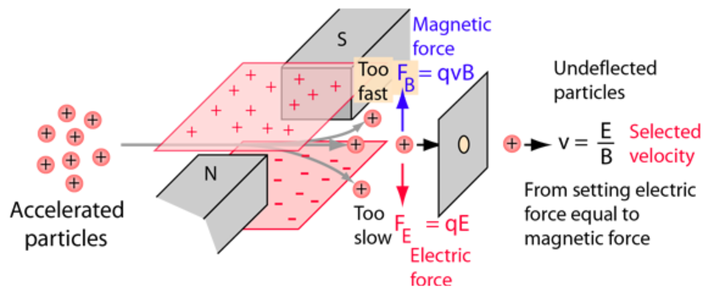
2. The magnetic force also cannot do work on the particle since the force and displacement vectors are always perpendicular.

3. This means kinetic energy stays the same since net work is 0. In other words speed will remain the same.

Aurora Borealis Charged ions follow a helical path towards the north pole. At such high speeds, they ionize the air and when the electrons recombine with the atoms, light is emitted. As it gets closer, the magnetic field gets stronger and speeds gets faster and the radius gets smaller.

6.4 Velocity Selector

In a small chamber, you have capacitor-like plates from the top and bottoms and a magnetic field coming from the sides. Say you have a regular xy plane on a piece of paper and the z axis is going into the page. So $\vec{E}$ is in the $-\hat{y}$ and $\vec{B}$ is in the $\hat{z}$. The particle is moving in the $\hat{x}$ direction.



In this setup we want $F_{net} = F_{byB}(\hat{y}) + F_{byE}(-\hat{y}) = 0$

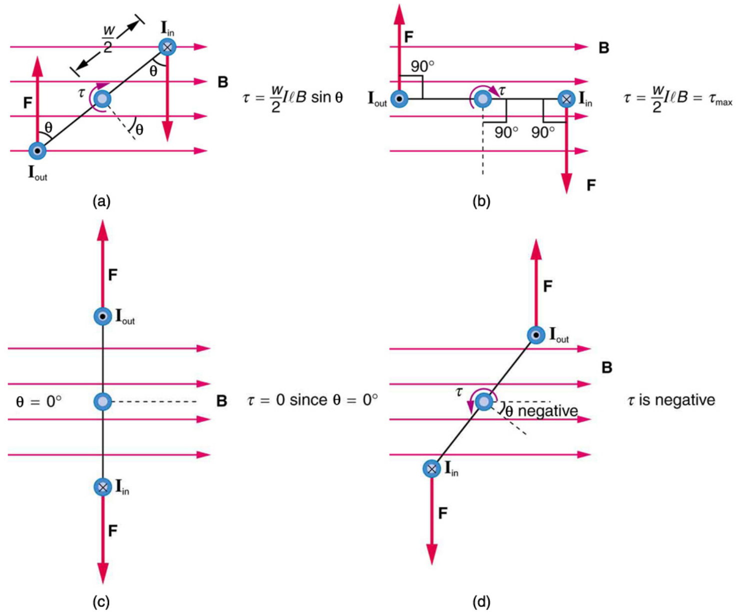
$$qvB = qE$$

$$v = \frac{E}{B}$$

Note that charge and mass play no real role here.

6.5 Torque on current loop

With a rectangular current loop placed in an external magnetic field. Let's say the rectangular too is a units long and b unit wide.



For one side we can write the torque as:

$$\tau_{one} = r \cdot F sin\theta = r \cdot (IlB) sin\theta = \frac{b}{2} \cdot IaBsin\theta$$

$$\tau = NIabBsin\theta = NIABsin(\theta)$$

$$\vec{\mu} = NI\vec{A}$$

This is quantity is known as the magnetic dipole moment of the coil.

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

You can always count that the forces are always up and down (in this scenario).

Electric dipole is $\vec{\tau} = \vec{p} \times \vec{E}$ and $U = -\vec{p} \cdot \vec{E}$. Similarly:

$$U = \int \tau d\theta = \int NIABsin\theta d\theta = -\mu Bcos\theta + C$$

$$U = -\mu Bcos\theta = -\vec{\mu} \cdot \vec{B}$$

6.6 Magnetic moment of a hydrogen atom

6.7 Electric Motors and Speakers

Brushed, brushless, induction motors

6.8 CRT and Milikan's

You have an emmiter. They are accelerated using a particle accelertor. Then using a velocity selector you filter the velocities. If you only put a magnetic field on, the path is curved. Using this fact, early experimenters found the ratio e/m.

$$evB = \frac{mv^2}{r}$$

$$\frac{e}{m} = \frac{v}{Br}$$

Using $v = \frac{E}{B}$

$$\frac{e}{m} = \frac{E}{B^2 r}$$

???

Robert A. Milikan found the charge of an electron. Tiny droplets of mineral oil caryying an electric charge were allowed to fall under gravity between two parallel plates. The electric field between the plates was adjusted until the drop was suspened in midair. The mass was determined by measuring the terminal velocity in the absense of an electric field. So $mg = qE$. After painstaking observations and analysis he found convincing evidence that any charge was an integral multiple of the smallest charge e which he discovered. Combined the e/m ratio found earlier, he derived the mass of an electron too.

6.9 Hall Effect

Moving charged particles in a wire will move to one side of the wire. Negative on one side and positive on the other. This creates another electric field across the wire.

$$F_H = eE_H = ev_d B$$
$$\varepsilon_H = E_H d = v_d B d$$

6.10 Mass Spectrometer

Veloctiy select on the accelerated particle.

$$v = \frac{E_A}{B_A}$$

Particle enters a magnetic chamber

$$qvB_M = \frac{mv^2}{r}$$
$$m = \frac{qB_M r}{v} = \frac{qB_M B_A r}{E}$$

7 Sources of Magnetic Fields

Just Ampere's Law and Biot-Savart Law

7.1 Ampere's Law

Analog to Gauss's Law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{encl}$$

Note this is on a 2d enclosed loop with some form of current flowing through it. Ampere's law is not limited to the magnetic field induced by the current. It also includes external magnetic fields. So it is important to note, the B field induced by the current already there, now add the external B field that will also change the current. The net change in both B and I will concur with Ampere's Law.

This law can be extended into the Maxwell-Ampere Law. But for circuits, it is good enough and we can ignore. ???

Nonuniform and nonzero???

7.2 Biot-Savart Law

Analog to Coulomb's Law

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{\vec{dl} \times \hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \hat{r}}{r^2}$$

7.3 Force between two Parallel Wires

One wire is just:

$$B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Using the Lorenz Equation:

$$F_2 = I_2 l_2 B_1$$

$$F_1 = I_1 l_1 B_2$$

We can substitute

$$F_2 = \frac{\mu_0 I_1 I_2 l_2}{2\pi d}$$

$$F_1 = \frac{\mu_0 I_2 I_1 l_1}{2\pi d}$$

Currents in the same direction attract. Currents in opposite directions repel. Reassure using the RHR.

7.4 Field Inside and outside a wire

Outside a wire is the usual:

$$B \cdot 2\pi R = \mu_0 I_{encl}$$

$$B = \frac{\mu_0 I_0}{2\pi R}$$

Inside the wire:

$$I_{encl} = I_0 \frac{\pi r^2}{\pi R^2} = I_0 \frac{r^2}{R^2}$$

$$B \cdot 2\pi r = \mu_0 I_0 \frac{r^2}{R^2}$$

$$B = \frac{\mu_0 I_0 r}{2\pi R^2}$$

See how (like before) B is a linear relationship inside the conductor and decays at $1/r$ outside.

7.5 Coaxial Cabel

7.6 Solenoid

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{encl}$$
$$Bl + 0 + 0 + 0 = \mu_0 N I$$
$$B = \mu_0 n I$$

where $n = N/l$

7.7 Toroid

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{encl}$$
$$B \cdot 2\pi r = \mu_0 N I$$
$$B = \frac{\mu_0 N I}{2\pi r}$$

7.8 Wire segment

7.9 Current Loop

7.10 Magnetic Field Due to a Moving Charge

domains.

8 Induction, Faraday's Law

Rule of Thumb: A changing magnetic field induces an emf.

8.1 Magnetic Flux

$$\Phi_B = \int \vec{B} \cdot \vec{dA}$$

8.2 Faraday's Law

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}$$

8.2.1 Faraday's Law in Circuits/Motors

Single loop:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Because it happens for every loop in a coil, for a N loop coil:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

8.3 Lenz's Law

Nature abhors change in flux

More formally: "The current induced in a circuit due to a change in a magnetic field is directed to oppose the change in flux and to exert a mechanical force which opposes the motion."

Contained inside $\varepsilon = -\frac{d\Phi_B}{dt}$ with the negative sign.

1. Area physically decreases
2. Flux decreases

Pointers:

1. We use Lenz's law to find conventional current
2. If flux is decreasing, induced magnetic field points with external field
3. If flux is increasing, induced magnetic field points in opposite direction as external field.
4. Better yet, imagine a simple current loop and a magnet at the bottom, where the magnet is your left hand and you are using the RHR in the right hand. Your right hand fingers curl against the direction of movement of your left hand. (Left goes up, Right fingers go down)
5. Seesaw
$$\varepsilon = -\frac{d\Phi_B}{dt}$$
6. Always remember that there are two magnetic fields. The external field whose flux is supposedly changing to induce current. The magnetic field produced by the induced current.
7. If wire is already carrying current, we add the currents.

8.3.1 Pulling Ring Example

8.3.2 Rod in a magnetic field

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{BdA}{dt} = \frac{Blvdt}{dt} = Blv$$

Basically shows us the emf created when any charged substance moves through a magnetic field. Because moving it at a certain speed creates flux.

8.3.3 Force on a Rod

$$\vec{l} \perp \vec{B} \tag{1}$$
$$F = IlB \tag{2}$$
$$I = \frac{\varepsilon}{R} = \frac{Blv}{R} \tag{3}$$
$$F = \frac{B^2 l^2 v}{R} \tag{4}$$
$$P_{ext} = Fv = \frac{B^2 l^2 v^2}{R} \tag{5}$$
$$P_R = I^2 R = \frac{B^2 l^2 v^2}{R} \tag{6}$$
$$\tag{7}$$

Where P_{ext} is the power from external environment that moves the rod and P_R is the power dissipated in the resistance .

8.4 Generators

Note $\omega = 2\pi f$ where f is cycles per seconds and ω is radians per second.

8.4.1 AC Generators

$$\varepsilon = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A} = -\frac{d}{dt} BAcos(\theta(t)) \tag{8}$$
$$\textbf{say } \theta(t) = \theta_0 + \omega t \tag{9}$$
$$\varepsilon = BA\omega \sin \omega t \tag{10}$$
$$\varepsilon = NBA\omega \sin \omega t \tag{11}$$
$$\textbf{where } \varepsilon_0 = NBA\omega \tag{12}$$

8.4.2 DC Generators

While you can just use a rectifier, there is an easier method with just a DC Generator. Communtators ???

8.4.3 Alternators

rotor is fet by current from the battery and is made to rotate by a belt from the engine. stator.

8.5 Back EMF and Counter Torque

If you turn a dc motor unplugged, you'll feel little rotational resistance. But if you complete the circuit and introduce a load to the dc motor, then rotational motion will result in a counter torque.

$$\varepsilon_{back} \propto \omega$$

accelerate? only angular speed??

9 LC Circuits

9.1 Self-Inductance

$$L = \frac{N\Phi_B}{I}$$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

9.2 Power

$$U = \frac{1}{2}LI^2$$

9.3 LR Circuits

Bottom line: break into Kirchoffs equations and solve Diff eq.

$$V_0 - IR - L\frac{dI}{dt}$$

$$\int \frac{dI}{V_0 - IR} = \int \frac{dt}{L}$$

$$-\frac{1}{R}\ln\left(\frac{V_0 - IR}{V_0}\right) = \frac{t}{L}$$

$$\frac{V_0 - IR}{V_0} = e^{-tR/L}$$

$$1 - \frac{IR}{V_0} = e^{-tR/L}$$

$$I = \frac{V_0}{R}(1 - e^{-tR/L})$$