eecs16a

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1 Relationships

$$V = \frac{dU}{dQ}$$

$$I = \frac{dQ}{dt}$$

$$P = IV = \frac{dU}{dt}$$

$$V = IR$$

- $1Volt = \frac{1Joule}{1Coulomb}$
- $1Amp = \frac{1Coulomb}{1second}$
- $1Watt = \frac{1Joule}{1second}$
- $1Joule = 1Coulomb * 1Volt$
- $1Coulomb = excessordeficitof 6.24 * 10^1 8e^-$
- $1Farad = 1Coulomb/1Volt$
- Permitivity: $1Farad/1meter$

1.1 In Depth Analysis of These Relationships

We can speculate that the motion of e^- mostly chaotic due to atomic collisions. When we introduce a voltage potential, we introduce a bias. In between collisions, they accelerate in a direction. You can imagine a mitochondrial membrane. The behavior of our ions are still chaotic, but the combination of the electrostatic forces create a clear bias.

The formal definition of current $\frac{dQ}{dt}$ through a given surface, such as a cross-sectional area of a wire. Note: current is the flow of any charge. They can be ions, electrons, or even neutral atoms. HyperText Current density

Believe it or not, pushing e^- through material requires work. Removing e^- from an hydrogen atom requires work. Change in potential energy is work.

Just like how two points away from Earth have a change in potential energy, two nodes in a circuit can have a change in electrical potential energy.

Note that $I = \frac{dQ}{dt}$ is solely the rate of change of e^- across a cross-sectional area. This is not analogous to 'velocity' or 'width of a pipe.' You can have a thin pipe with a lot of e^- passing through, giving you a large current. You can have a fat pipe with barely any e^- passing through, giving you a small current. You can also have a lot of e^- passing through at a large cross-sectional area at 'low velocity' and still have a large current.

$$Q \propto \times elementarycharge$$
$$Q \propto \text{number of } e^-$$

$$\begin{aligned} U &= QV && \text{our voltage definition} \\ U &\propto Q \\ U &\propto V \end{aligned}$$

Note that U represents the electric potential energy. Just like gravity, U is the energy stored in the electric field. Try to think of it as two charged particles being attracted or repelled from one another, just like two objects in space (at least the attracting part).

1.2 Derivations: Charges

Charles Augustin Coulomb experimentally noticed properties between charged particles. These experimental relationships, he captured in what is known as Coulomb's Law.

$$\begin{aligned} F_e &= k\frac{q_1 q_2}{r^2} && k = \frac{1}{4\pi\epsilon_0} \\ \vec{F_e} &= k\frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^2}\vec{u} && \text{Vector notation} \\ \vec{u} &= \frac{\vec{r_1} - \vec{r_2}}{|\vec{r_1} - \vec{r_2}|} \end{aligned}$$

$$\begin{aligned} F_e &= qE \\ E_{q_1} &= \frac{F_{q_1 \to q_2}}{q2} = k\frac{q_1}{|\vec{r_1} - \vec{r_2}|^2}\hat{r} && \text{Electric Field in respect to one reference particle} \end{aligned}$$

The Electric Field is the Coulomb force per unit charge. The Coulomb force is the actual force field in the system. The electric field is the field for 1 Coulomb.

1.3 Derivations: Electric Potential

Coulomb's law is similar to Newton's Law of Gravitation. Note: that electrostatic forces depend on charge while gravitation depends on mass; gravitation only attracts while electrostatic attracts and repels; a e^- has both gravitational force and electrostatic force, but the electrostatic force is way stronger for such small particles.

Just like there is potential energy when two objects are held away from each other in space, there is potential energy when two charges are held two points in space. Assume one dimensional non-wire system. Assume conservative electric field. Let there be two charged particles floating in vacuum on an axis, let r be the distance in between:

$$\begin{aligned}
\Delta U &= \int_{r_1}^{r_2} \vec{F_e} \cdot \vec{dr} && \text{Analogous to gravitational potential energy} \\
&= \int_{r_1}^{r_2} k\frac{q_1 q_2}{r^2}\, dr \cos 0 \\
&= kq_1q_2 \left(-\frac{1}{r}\right)_{r_1}^{r_2} \\
&= kq_1q_2 \left(\frac{1}{r_1} - \frac{1}{r_2}\right) \\
&= -\left[\frac{kq_1q_2}{r_2} - \frac{kq_1q_2}{r_1}\right] \\
V_{q_1} &= -\frac{\Delta U}{q_2} = kq_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right) && \text{Voltage is the integral of the electric field} \\
&= \frac{-\int E_{q1}\, dr}{q_2}
\end{aligned}$$

$$\Delta V = \frac{\Delta PE}{q} \tag{1}$$

Voltage is the potential energy per unit charge. So it is the PE in 1 Coulomb (or other units). PE is just the potential energy within the system. Both still grasp the idea of energy. Voltage is in J/C while PE is just in J. A car battery and a flashlight battery may have the same the voltage, but not the same potential energy.

Just as two objects in a viscous fluid attract slowly, two charged particles in a resistive medium can attract at lower currents. Hence, $V = IR$

Under the Maxwell-Faraday Equation, we can assume our vector field is conservative if the magnetic field remains constant over time.

$$\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

Note: Φ and V_e are interchangeable. Voltage is the line integral of the static electric field. Let's assume it is conservative, so any path gets us the same voltage difference. Equally, the electric field is the gradient of the voltage. This means the electric field points "downwards" toward lower voltages. Our derived Maxwell equation shows that the divergence of the electric field is directly related to the total charge density at that point.

$$\begin{aligned} V_e &= -\int_C \vec{E} \cdot ds \\ E &= -\nabla V_e \\ &\to \nabla \cdot E = -\nabla^2 V_e \\ &\to \nabla \cdot E = \frac{\rho}{\epsilon_0} \end{aligned}$$

1.4 Derivations: Current

Q is a unit of charge. Q of a group of e^- is dependent on two things: strength of the e^- charge and the amount of e^-.

One coulomb is hence 6.24×10^{18} electrons.

The faraday unit of charge is elementary charge $\times$ one mole. AKA the charge of one electron $\times$ one mole.

$$F = e \times N_A$$
$$F = 1.602 \times 10^{-19} C \times 6.022 \times 10^{24} moles$$

Note that current is dependent on a few things: strength of charge, number of these charged particles, drift speed, cross sectional area.

This is another common way to define current in terms of a wire. It is drift speed $\times$ cross sectional area $\times$ average charge $\times$ discrete number of charged particles

$$I = nqAv$$

This is the most broad and general way of writing current. Note: depending on how you define your graph / normal vectors, you'd either get positive or negative of the result. Note: dot product shows us how overlapped.

$$\text{Recall: } SA = \iint_S F(\vec{r}(u,v)) ||\vec{r_u} \times \vec{r_v}|| du dv$$

$$I = \oint\!\!\!\oint_S \vec{J} \cdot d\vec{S} \qquad\qquad d\vec{S} = \hat{n} dS$$

Current density is the amount of current while accounting for cross sectional area. So only number of charged particles × charge of the particles × drift velocity.

$$J = \frac{I}{A} = nqv$$

Charge Density is the amount of charge in a given space.

$$\rho = \frac{Q}{V}$$

$$\frac{\rho}{\epsilon_0} = \nabla \cdot \vec{E}$$

$$\begin{aligned} J &= \rho v \\ &= \frac{Q}{V}\frac{\Delta s}{\Delta t} \\ &= \frac{\Delta s}{\Delta s V}\frac{Q \Delta s}{\Delta t} \\ &= \frac{1}{V}\frac{\Delta Q}{\Delta t} \\ &= I/A \end{aligned}$$

There is the water-pipe model of current, where fluid mechanics translates over to electrons. Here we find the "mass continuity equation" in fluid mechanics.

$$\begin{aligned} I &= JA \\ &= \rho v A \\ \rho_1 v_1 A_1 &= \rho_2 v_2 A_2 && \text{Mass Continuity Equation} \end{aligned}$$

2 Resistors

All the e^- collides with atoms in the material, causing atoms to vibrate, generating heat.

$$R = \rho \frac{L}{A}$$

Notice: $L \to \infty, R \to \infty$
$A \to \infty, R \to 0$

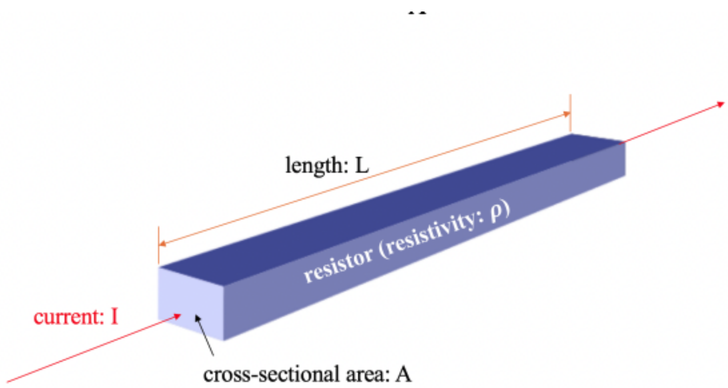


Figure 1: Imagine the electrons pushing through the material

2.1 Resistor Simplifications Derivation

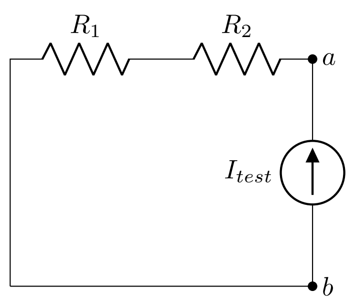


Figure 2: The "black box" are the two resistors. Since there are no independent sources, we plug in a current source because when V or I is zero, we do not extract the IV relationship.

$$
\begin{aligned}
V_{test} &= V_{R_1} + V_{R_2} && (KVL) \\
V_{test} &= i_{R_1} R_1 + i_{R_2} R_2 && I_{test} = i_{R_1} = i_{R_2} \\
V_{test} &= I_{test}(R_1 + R_2) && \\
R_{th} &= \frac{I_{test}(R_1 + R_2)}{I_{test}} && R_{th} = \frac{V_{test}}{I_{test}} \\
R_{th} &= R_1 + R_2 &&
\end{aligned}
$$

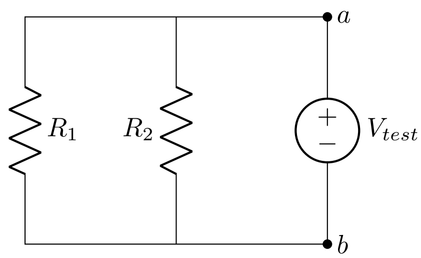


Figure 3: We plug in a voltage source because when V or I is zero, we do not extract the IV relationship.

$$
\begin{aligned}
I_{test} &= i_{R_1} + i_{R_2} \\
I_{test} &= \frac{V_{R_1}}{R_1} + \frac{V_{R_2}}{R_2} \qquad\qquad V_{test} = V_{R_1} = V_{R_2} \\
I_{test} &= V_{test}(\frac{1}{R_1} + \frac{1}{R_2}) \\
R_{th} &= V_{test}\frac{1}{V_{test}(\frac{1}{R_1} + \frac{1}{R_2})} \\
R_{th} &= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}
\end{aligned}
$$

Note that the equivalent resistance is lower than the individual resistors. This makes sense as we add more and more resistors in parallel, we lower the overall resistance to current.

3 Resistor Touchscreen

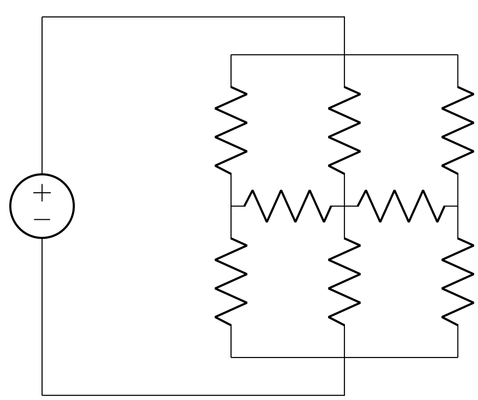


Figure 4: You are used to seeing this where the top and bottom are nodes

Let our resistor touchscreen be two resistor meshes, which are separate resistive material that can bend to touch each other. In figure 6, the bottom mesh has current flowing through a bunch of voltage dividers. The top mesh probes the points on the bottom mesh. The top mesh is connected to our voltmeter. Since voltmeter is an open circuit, no current flows through and it only tells us the potential difference of our probed node in respect to ground.

When a touch is sensed (by alternating between meshes to check for nonzero potential differences), our Launchpad reads the voltmeter and then makes the bottom mesh the "voltmeter" and the top mesh circuit of voltage divider. Hence, we attain a second potential difference value.

Note: another possibility is to alternate the voltage source on the bottom mesh vertically and horizontally and have the top mesh only to measure voltage.

Figure 5: Resistor touchscreen layout

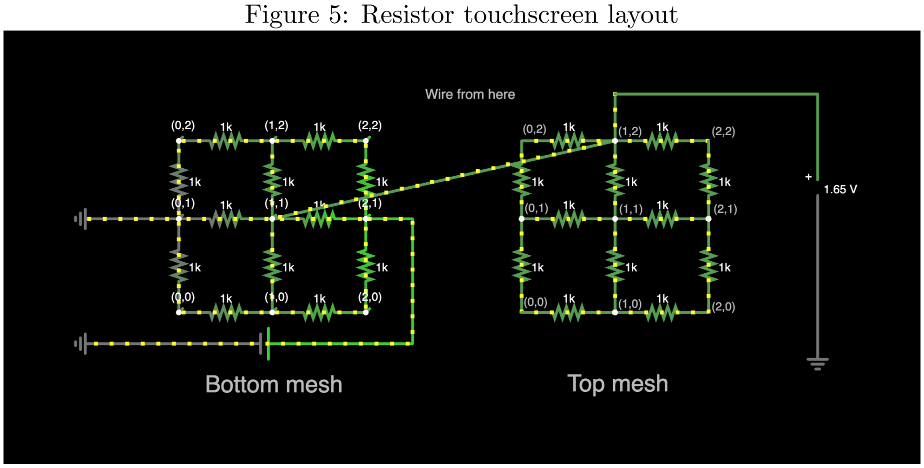
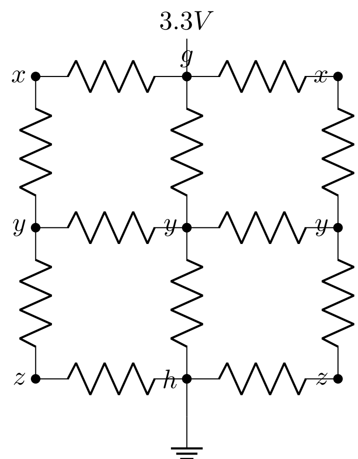
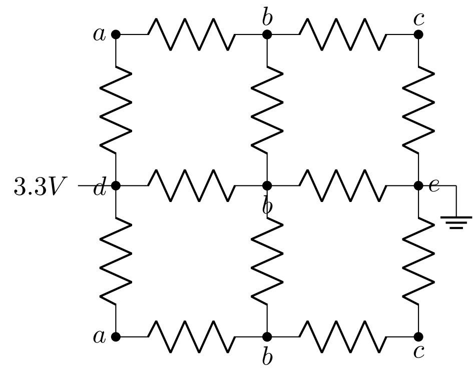


Figure 6: Note: the 'wire' touching the two meshes would technically be at the same point. The diagram above shows that it doesn't matter where you probe with the top mesh, you'd get the same reading.

How does this new configuration, without a solid node on the top or bottom, get us an unique coordinate?





$$
\begin{aligned}
y &= 3.3V \frac{R}{R+R} & \qquad y &= y - 0 \\
&= 3.3V \frac{R+R}{R+R+R+R} \\
&= 1.65V \\
x &= (x-y) + y & \qquad y' &= g - y = 3.3V \frac{R}{R+R} \\
&= y' \frac{R}{R+R} + y \\
&= 2.475V \\
z &= y \frac{R}{R+R} \\
&= .825V \\
y &= b \\
x &= a \\
z &= c
\end{aligned}
$$

There are repeated values in our diagram such as x, y, z. By flipping the circuit's voltage source, we get unique positions from the voltage differences.

- (x, a)
- (y, d)
- (z, a)
- (g, b)
- (y, b)
- (h, b)
- (x, c)
- (y, e)
- (z, c)

4 Capacitors

$$Q = CV \tag{2}$$

Capacitance, the amount it can hold, is usually constant in our circuits. Q is the amount of charge on one plate (it would be zero if it were on two). Voltage is work that can shove electrons onto a plate.

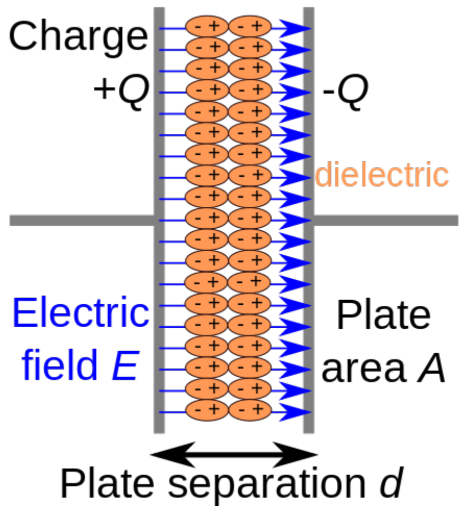
$$\frac{dQ}{dt} = C\frac{dV}{dt}$$
$$I = C\frac{dV}{dt} \tag{3}$$

Current has a direct relationship with the rate of change of voltage. This implies that current on the wire above only exists when voltage across the capacitor is changing.

$$C = \epsilon\frac{A}{d} \tag{4}$$

Capacitance is determined by physical properties. For parallel-plated capacitors, we model the relationship in (4). Epsilon is the permittivity. Area is the area of overlap. Note: even if you have a thick upper plate, what only matters is the surface area (where the e^- will lie.)
$A \to \infty, C \to \infty$
$d \to \infty, C \to 0$

Polarized Caps increase the capacitance through a polarized dielectric. Imagine two plates separated by a vacuum (has permittivity of ϵ_0). Imagine two plates separated by a polarized substance like water. Polarized molecules have an unequal distribution of charge. Hence, they will begin to orient themselves. This orientation allows more charge to flow on the plates as they are pulled in more by the polarized molecules.



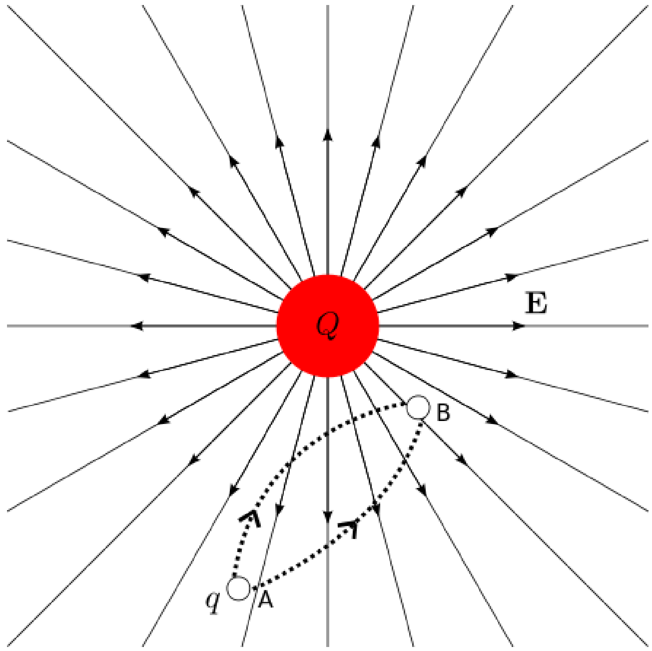
4.1 More Derivations for the Curious Minded

Because Electric field is constant throughout the space between the plates as vectors cancel out.

$$E \times d = V$$

More specifically:

$$\begin{aligned} V_{AB} &= V(r_B) - V(r_A) \\ &= -\int_0^{r_B} E dr - (-\int_0^{r_A} E dr) \\ &= -\int_{r_A}^{r_B} E dr \end{aligned}$$



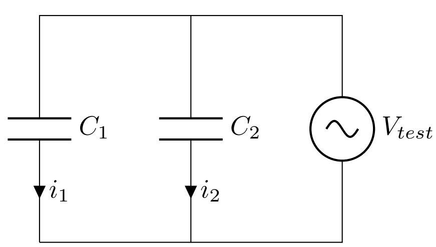
The electric potential energy stored between two plates are $U = \frac{CV^2}{2}$.

$$dE = V_c dQ$$
$$dQ = C dV_c$$
$$\int_0^{E_{eq}} = \int_0^{V_{eq}} V_c(C) dV_c$$
$$E_{eq} = \frac{1}{2} C V_{eq}^2$$

4.2 Capacitor Simplifications Derivations

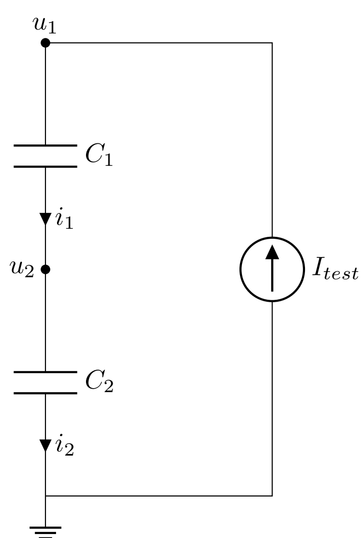
$$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}}$$

In parallel:



$$
\begin{aligned}
I_{c_1} &= C_1 \frac{dV_{c_1}}{dt} \\
I_{c_2} &= C_2 \frac{dV_{c_2}}{dt} \\
I_{test} &= I_{c_1} + I_{c2} \\
I_{test} &= (C_1 + C_2) \frac{dV_{test}}{dt} \\
C_{eq} &= C_1 + C_2
\end{aligned}
$$

In series:

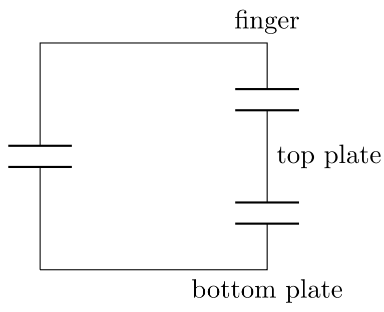


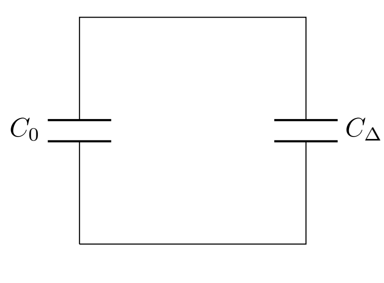
$$\begin{aligned}
I_{test} &= I_{c_1} = I_{c_2} \\
I_{c_1} &= C_1\frac{V_{c_1}}{dt} = C_1\frac{d(u_1-u_2)}{dt} \\
I_{c_2} &= C_2\frac{V_{c_2}}{dt} = C_2\frac{du_2}{dt} \\
&\rightarrow \frac{I_{c_2}}{C_2} = \frac{du_2}{dt} && \text{Voltage Defs} \\
I_{c_1} &= C_1\frac{du_1}{dt} - C_1\frac{I_{c_2}}{C_2} \\
I_{test} &= C_1\frac{dV_{test}}{dt} - \frac{C_1}{C_2}I_{test} \\
(1+\frac{C_1}{C_2})I_{test} &= C_1\frac{dV_{test}}{dt} \\
I_{test} &= \frac{C_1}{1+\frac{C_1}{C_2}}\frac{dV_{test}}{dt} && I_{test} = C_{eq}\frac{dV_{test}}{dt} \\
C_{eq} &= \frac{C_1C_2}{C_1+C_2}
\end{aligned}$$

Notice that parallel operator means that the equivalent capacitance is lower than either of the individual capacitors. This makes intuitive sense because you basically made the capacitors less effective by widening the distance.

Note that this forms a capacitor based voltage divider, where $\frac{du_2}{dt} = \frac{C_1}{C_1+C_2}\frac{du_1}{dt}$.

4.3 Capacitor Touchscreen





When your finger touches, you create C_Δ, which means the overall capacitance from top plate to bottom plate increases. Now let's add that with with the following circuit.

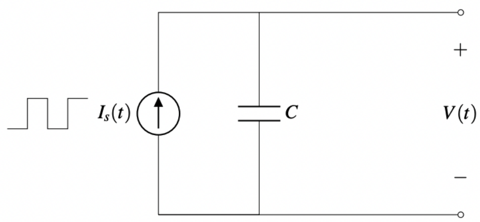
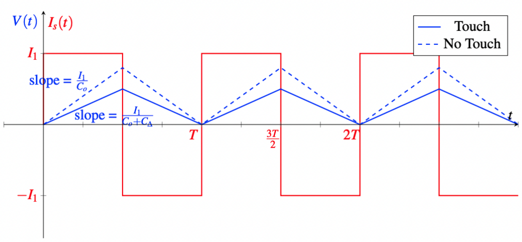


Figure 7: We use a square wave current source

We can use the equation $V = \frac{I}{C}(t - t_0) + V(t_0)$ because the current is constant for discrete periods. Notice that when we increase the capacitance of our capacitor in 7, we will lower C in our linear equation. Hence our V(t) will have a flatter slope. We can take advantage of this flatter slop to detect touch.



The point in between the highest point of no-touch and touch give us a good boundary. We can create an if-then statement via a comparator to check when it is above our boundary or below. We get the resulting square wave when our V(t) passes the boundary. Hence we know there is no touch.

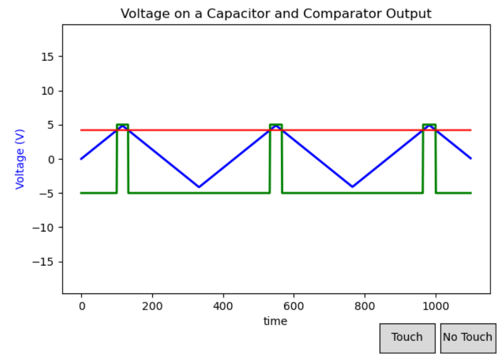


Figure 8: The red line is our V_{ref} or boundary. The green line is our comparator detecting our signal passing V_{ref}

4.4 Charge Sharing Algorithm

$$Q_u^{\Phi_1} = Q_u^{\Phi_2} \tag{5}$$

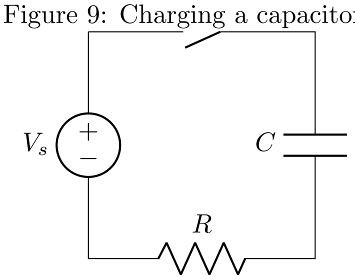
1. Label the voltages across all the capacitors. Show the polarity of the capacitors. Just remain consistent throughtout the phases.
2. Draw the equivalent phases circuits
3. Identify floating nodes. These are nodes in which no charge can escape. For example, a node that is only connected to a plate(s).
4. **Find** $Q_u^{\Phi_1}$. For each floating node in 2nd phase, we find the total charge of this node by examining the capacitor plates of steady state in the previous phase. Careful: The plate marked with the "$-$" sign will have $Q = -CV_C$ and the plate marked with the "+" sign. Also note that you are fundamentally finding the amount of storage the capacitor can store. (Remember to use the voltage difference across the capacitor)
5. **Find** $Q_u^{\Phi_2}$. Find the total charge of the floating nodes in the steady state of phase 2. (Remember to use the voltage difference across the capacitor)
6. Use the conservation of floating node charges to find node voltages, etc.

4.5 Capacitor Derivations

Assuming we start at a completely discharged capacitor and add charge up to any point int time. The potential energy at that point (in respect to our discharged state) is derived below. U and V can be equilibrium values.

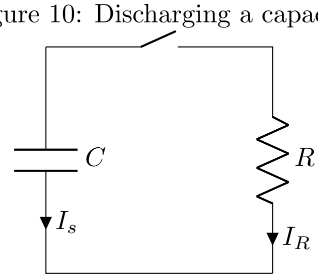
$$
\begin{aligned}
dU &= V_C dQ && \text{To store an additional dQ charge} \\
&\rightarrow dQ = C dV_C && \text{Q = CV} \\
\int_0^U dU &= C \int_0^V V_C dV_C \\
U &= \frac{1}{2} C V^2
\end{aligned}
$$

Figure 9: Charging a capacitor



$$
\begin{aligned}
V_s - V_{cap} - V_R &= 0 && \text{KVL} \\
V_s - \frac{Q}{C} - I_R R &= 0 && \text{With resistor, we get the FODE} \\
\rightarrow -R\frac{dQ}{dt} &= \frac{Q - CV_s}{C} && \text{Current is constant throughout} \\
\frac{1}{Q - CV_s} dQ &= -\frac{1}{RC} dt \\
\int_0^{Q(t)} \frac{1}{Q - CV_s} \, dQ &= \int_0^t -\frac{1}{RC} \, dt \\
\ln \frac{CV_s - Q(t)}{CV_s} &= -\frac{t}{RC} \\
Q(t) &= CV_s(1 - e^{-t/RC})
\end{aligned}
$$

Figure 10: Discharging a capacitor



$$
\begin{aligned}
V_{cap} - V_R &= 0 \\
\frac{Q}{C} - I_R R &= 0 \\
\frac{Q}{C} + R\frac{dQ}{dt} &= 0 \\
\frac{dQ}{Q} &= -\frac{dt}{RC} \\
\int_{Q(0)}^{Q(t)} \frac{dQ}{Q} &= -\int_0^t \frac{dt}{RC} \\
\ln\frac{Q(t)}{Q(0)} &= -\frac{t}{RC} \\
Q(t) &= Q(0)e^{-t/RC}
\end{aligned}
$$

4.6 Misc.

$$Z = R + jX_c$$

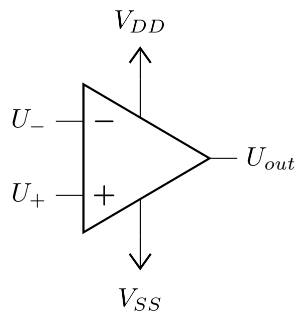
Impedance is all about resistance to AC signals. Note that it includes both resistance and reactance. For example, a resistor, inductor, and capacitor can affect the AC signal in different ways.

$$X_C = \frac{1}{j\omega C}$$

$$X_L = j\omega L$$

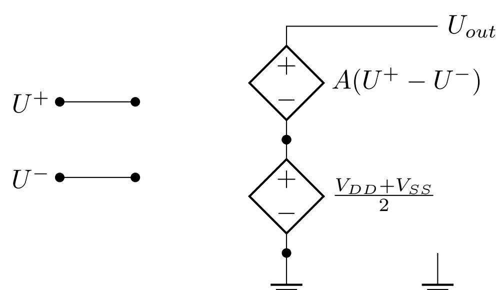
- Smoothing caps smooth out noise in signal (Ex: rectifier)
- Coupling caps rid of low frequency. Basically, only allows AC signals and blocks out DC.
- Bypass caps smooth noise in our DC voltage source.

5 Op Amps



$$U_{out} = \begin{cases} V_{dd} \\ A(U_+ - U_-) + \frac{V_{dd}+V_{ss}}{2} \\ V_{ss} \end{cases}$$

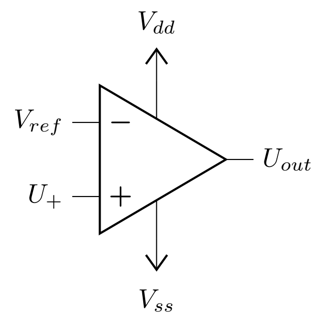
Sometimes circuit analysis requires us to use a different model. Note that U^+ and U^- are open circuits, hence no current will flow. We only care about the change in voltage.



5.0.1 Comparators

Comparators capture the if-then logic of our signals. Basically, if $V_{in} > V_{ref}$ give 3.3V else 0V, etc.

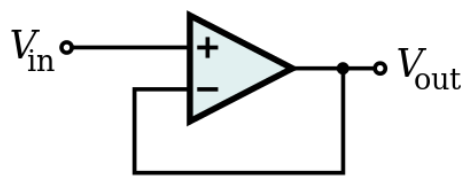
We can do so by setting amplification super high and use a V_{ref} to compare to. So if U^+ is just slightly higher than V_{ref} we hit the upper rail. If slightly lower, we hit the lower rail.



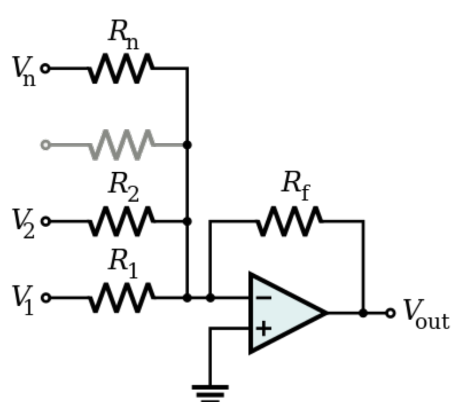
5.0.2 Buffers

Buffers allow use to create modular design of electronics. It can separate our circuit without having to worry about loading (connecting low impedence to high impedence, etc).

Note that $V_{in} = V_{out}$ and so the voltage signal is preserved. We've only changed the current.

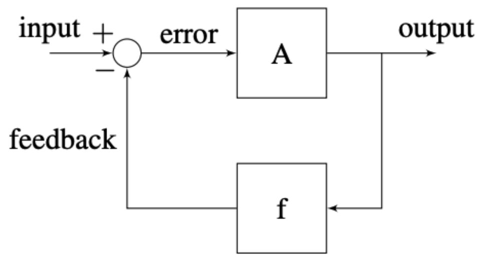


5.0.3 Summing Amplifier



5.1 Negative Feedback

You can think of negative feedback as though balancing an umbrella on our hand. You apply a counteracting force.



For simplicity, imagine our input is constant. At each discrete time step, we will apply a correcting force. In this case, each time you loop around you'll apply a correction.

$$S_{err} = S_{in} - S_{fb}$$
$$S_{out} = A * S_{err}$$
$$S_{fb} = f * S_{out}$$

With some algebraic manipulations, we get:

$$\frac{S_{out}}{S_{in}} = \frac{A}{1 + Af}$$
$$\frac{S_{out}}{S_{in}} = \frac{1}{f} \qquad \text{as } A \to \infty$$

This means that for ideal op amps, we control the gain with the f block.

It is worth noting that Voltage Gain ($\frac{V_{out}}{V_{in}}$) and A are different. 'A' is the device's inherent amplification factor, which is fixed. 'A_v' is the gain, which is not fixed.

The f block is reponsible for changing voltage. Often times, f block is just passive components, so our v_{fb} be either the output voltage or less. In our equations: $f \leq 1$ or $V_{fb} \leq V_{out}$. Then 'A' is just the maximal gain. We can tune down the amplification, going down from the maximal amount of amplification, via negative feedback.

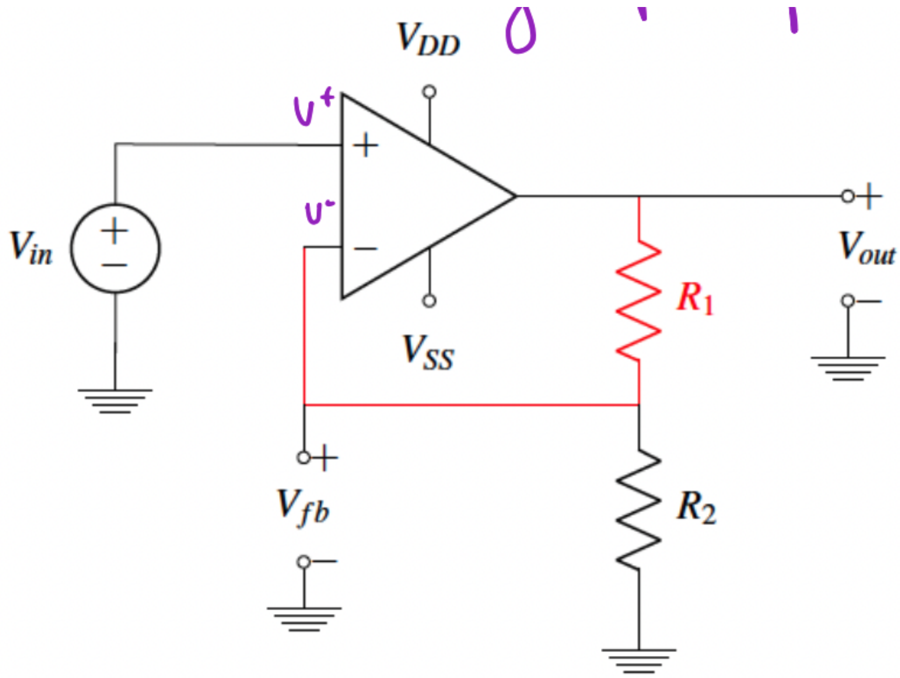


Figure 11: Generic Negative Feedback for an Ideal Op Amp

We know the following:

$$
\begin{aligned}
V_{err} &= V_{in} - V_{fb} \\
V_{out} &= AV_{err} \\
V_{fb} &= \frac{R_2}{R_1 + R_2} V_{out}
\end{aligned}
$$

So we can derive the following:

$$
\begin{aligned}
V_{out} &= A(V_{in} - V_{fb}) \\
V_{out} &= A(V_{in} - fV_{out}) \\
V_{out}(1 + Af) &= AV_{in} \\
A_v &= \frac{V_{out}}{V_{in}} = \frac{A}{1 + Af} \\
f &= \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2} \qquad \text{as } A \to \infty
\end{aligned}
$$

So we can derive the following:

$$
\begin{aligned}
V_{out} &= AV_{err} \\
V_{err} &= \frac{1}{A}\frac{A}{1+Af}V_{in} \\
V_{err} &= \frac{V_{in}}{1+Af} \\
V_{err} &= 0 && \text{as A} \to \infty \\
U^+ &= U^-
\end{aligned}
$$

5.2 Golden Rules

1. $I^+ = I^- = 0$ for all op amps
2. $U^+ = U^-$ only for NF, $A \to \infty$

The inputs on an op amp are open circuits, so we know no current goes through. Voltage can change though.

For intuition about why GR2 is true, imagine an unity buffer op amp with an amplification factor of 10^6 and $V_{DD} = -V_{SS}$. Let's say that $V_{in} = U^+ = 5V$. We know that $V_{out} = U^-$. We know that $A(U^+ - U^-)$ so $10^6(5 - V_{out}) = V_{out}$. In order for this to be true, $V_{out} = 4.999995V$

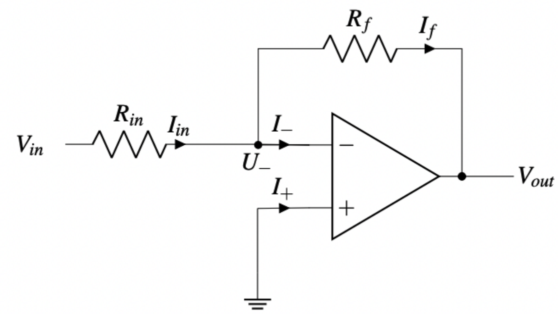
5.3 Testing Negative Feedback

1. Zero out all independent sources. Replace current sources with open circuits and voltage sources with wires. So V_{in} and V_{out} becomes connected to ground (usually).
2. Wiggle the output and check the loop. Increase the voltage of output slightly by making V_{out} an open circuit. Trace backwards. How does V_{err} change? (This is $U^+ - U^-$) Remember that all our voltage signals are relative to ground. If the V_{err} less, then we are in negative feedback. If the expression is greater, we are in positive feedback.

Short Cut:

- **GR2:** $U^+ = U^-$ find node potential relationships.
- **KCL** write KCL equations for U^+ and U^-. Write in terms of node potentials. Cancel via GR1.

5.4 Inverting Op Amp



$$
\begin{aligned}
I_{in} &= I_f && \text{KCL and GR1} \\
U^+ &= U^- = 0 && \text{GR2 (NF and ideal)} \\
I_{in} &= \frac{V_{in} - U^-}{R_{in}} \\
I_f &= \frac{U^- - V_{out}}{R_f} \\
\frac{V_{in}}{R_{in}} &= -\frac{V_{out}}{R_f} \\
A_v &= -\frac{R_f}{R_{in}}
\end{aligned}
$$

Note that the gain inverts the signal. For actual higher gain, $R_{in} \leq R_f$.

5.5 Buffer Op Amp

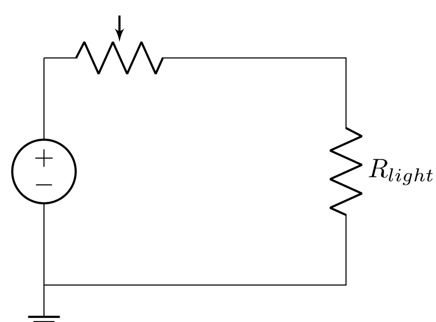


Figure 12: Notice that changing our pot, changes the voltage that our load gets. If we had a speaker or a lightbulb, this would cause problems.

We can prevent loading of the battery by introducing an unity buffer. Loading is when resistor load affects previous parts of the circuit.

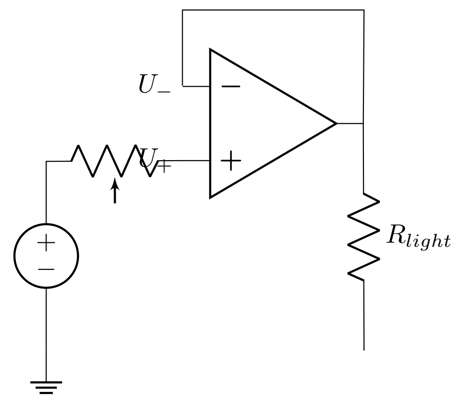


Figure 13: Now, changing our potentiometer doesn't affect the voltage over the light. Voltage signal is preserved. Impedance can be ignored.