Problems for Integration Bee

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1 Problems

1.1 U-sub

$$\int \frac{\sqrt{x}}{x^3+1} dx \tag{1}$$

$$u = x^{3/2}$$
$$\int \frac{\sqrt{x}}{x^3+1} dx = \frac{2}{3} \arctan(x^{3/2})$$

1.2 Some insight

$$\int \sin(2x)(\sin(x)+1)^{2023} dx \tag{2}$$

$$= \int 2\sin(x)\cos(x)(\sin(x)+1)^{2023} dx$$
$$= 2\int u(u+1)^{2023} du \quad u = \sin(x)$$
$$= 2\int (u+1-1)(u+1)^{2023} du \qquad \text{another way is Integration by Parts}$$
$$= 2\int (u+1)^{2024} - (u+1)^{2023} du$$
$$= \frac{2(u+1)^{2025}}{2025} - \frac{2(u+1)^{2024}}{2024}$$
$$= \frac{2(\sin(x)+1)^{2025}}{2025} - \frac{2(\sin(x)+1)^{2024}}{2024}$$

1.3 Exponent rules

$$\int \log_{10}(2^x)dx \tag{3}$$

$$= \int \frac{\ln(2^x)}{\ln(10)}$$
$$= \int \frac{\ln(e^{x\ln(2)})}{\ln(10)}$$
$$= \int \frac{x\ln(2))}{\ln(10)}$$
$$= \frac{\ln(2)}{\ln(10)} \int x dx$$
$$= \frac{x^2\ln(2)}{2\ln(10)}$$

or u-sub

$$= \int \frac{\ln(2^x)}{\ln(10)} dx \quad u = \ln(2^x)$$
$$= \frac{1}{\ln(2)\ln(10)} \int u du \quad du = \frac{2^x\ln(2)}{2^x} dx = \ln(2)dx$$
$$= \frac{1}{\ln(2)\ln(10)} \frac{u^2}{2}$$
$$= \frac{\ln^2(2^x)}{\ln(4)\ln(10)}$$

1.4 Convert to polar

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2-y^2} dx dy \tag{4}$$

$$= \int_0^{2\pi} \int_0^{\infty} e^{-r^2} r dr d\theta \quad u = -r^2 \quad du = -2r dr$$
$$= \frac{-1}{2} \int_0^{2\pi} \int_0^{-\infty} e^u du d\theta$$
$$= \frac{-1}{2}(-1) \int_0^{2\pi} d\theta$$
$$= \pi$$

1.5 Series Series

$$\int_{-1/6}^{1/6} 1 + 2x + 3x^2 + 4x^3 + ...dx \tag{5}$$

$$\begin{aligned}
&= x + x^2 + x^3 + x^4 + ...\big|_{-1/6}^{1/6} \\
&= \sum_{n=1}^{\infty} x^n \Bigg|_{-1/6}^{1/6} \\
&= \left(\frac{1}{1-1/6} - \frac{1}{1+1/6}\right) \\
&= \left(\frac{1}{5/6} - \frac{1}{7/6}\right) \\
&= \left(\frac{6}{5} - \frac{6}{7}\right) \\
&= \left(\frac{42-30}{35}\right) \\
&= \frac{12}{35}
\end{aligned}$$

1.6 Complex Identities

$$\int \cosh x e^x dx \tag{6}$$

Use hyperbolic identity $\cosh x = \frac{1}{2}(e^x + e^{-x})$

$$\begin{aligned}
&= \frac{1}{2}\int e^{2x} + 1dx \\
&= \frac{1}{2}(\frac{1}{2}e^{2x} + x) \\
&= \frac{1}{4}e^{2x} + \frac{x}{2}
\end{aligned}$$

1.7 Some trig

$$\int \frac{1}{1+\sin(x)} dx \tag{7}$$

$$
\begin{aligned}
&= \int \frac{1}{1+\sin(x)} \frac{1-\sin(x)}{1-\sin(x)} dx \\
&= \int \frac{1-\sin(x)}{1-\sin^2(x)} dx \\
&= \int \frac{1-\sin(x)}{\cos^2(x)} dx \\
&= \int \sec^2(x) - \sec(x)\tan(x) dx \\
&= \tan(x) - \sec(x)
\end{aligned}
$$

1.8 Int by Parts

$$\int \frac{\sin(x)}{x} + \ln(x)\cos(x) dx \tag{8}$$

$$
\begin{aligned}
&= \int \frac{\sin(x)}{x} dx + \int \ln(x)\cos(x) dx \\
&= \int \frac{\sin(x)}{x} dx + \left(\ln(x)\sin(x) - \int \frac{\sin(x)}{x} dx \right) \\
&= \ln(x)\sin(x)
\end{aligned}
$$

1.9 Absolute Values

$$\int_{-1}^{\infty} e^{-|\pi x|} dx \tag{9}$$

$$
\begin{aligned}
&= \int_0^{\infty} e^{-\pi x} dx + \int_{-1}^{0} e^{\pi x} dx \\
&= -\frac{1}{\pi} \left(e^{-\pi x} \right)_0^{\infty} + \frac{1}{\pi} \left(e^{\pi x} \right)_{-1}^{0} \\
&= -\frac{1}{\pi} (0-1) + \frac{1}{\pi} \left(1 - e^{-\pi} \right) \\
&= \frac{2}{\pi} - \frac{1}{\pi e^{\pi}}
\end{aligned}
$$

1.10 More tricky u-sub

$$\int \frac{1}{e^x + e^{-x}} dx \tag{10}$$

$$= \int \frac{1}{e^x + e^{-x}} dx$$
$$= \int \frac{e^x}{e^{2x} + 1} dx \quad u = e^x$$
$$= \int \frac{1}{u^2 + 1} dx \quad u = e^x$$
$$= \arctan(e^x)$$

1.11 Switching X,Y

$$\int_1^8 \int_1^{\sqrt[3]{x}} \frac{x}{448y - y^7 - 26} dydx \tag{11}$$

$$= \int_1^8 \int_1^{\sqrt[3]{x}} \frac{x}{448y - y^7 - 26} dydx$$
$$= \int_1^2 \int_{y^3}^8 \frac{x}{448y - y^7 - 26} dxdy$$
$$= \frac{1}{2} \int_1^2 \frac{8^2 - y^6}{448y - y^7 - 26} dy \quad u = 448y - y^7 - 26$$
$$= 14 \ln(2)$$

2 Working on

2.1 Under the integral sign

$$\int \tag{12}$$

$$\int$$

2.2 Taylor series

$$\int_{-\pi/6}^{\pi/6} \frac{1}{(1 - x^3)^3} dx \tag{13}$$

$$= x + x^2 + x^3 + x^4 + ...$$
$$= \sum_{n=1}^{\infty} x^n$$
$$= \left(\frac{1}{1 - \pi/6} - \frac{1}{1 + \pi/6} \right)$$

2.3 Convert to Imaginary

$$\int \sin(x)dx \tag{14}$$

$$Im[e^{i\omega t}]$$

2.4 Contour Integration

$$\int \frac{1}{3 - 4\cos(x)}dx \tag{15}$$

$$\int$$

2.5 Odd

$$\int_{-5}^{5} \sin^3(x)\tan^2(x)dx \tag{16}$$

$$= 0$$

$$\begin{aligned}
&\max_\theta LL \\
&\quad = \max_\theta \sum log(f) \\
&\quad = \max_\theta \sum log(\binom{n_i}{m_i} f_i^m (1-f)^{n_i - m_i} \\
&\quad = \max_\theta \sum log(\binom{n_i}{m_i} f_i^m + log(1-f)^{n_i - m_i} \\
&\quad = \max_\theta \sum m_i \cdot log(\binom{n_i}{m_i} f + (n_i - m_i) \cdot log(1-f)
\end{aligned}$$

$$\frac{dll}{dw_0} = \frac{dll}{df}\frac{df}{dz}\frac{\partial z}{\partial w_0} = \frac{m_i}{\sigma} - \frac{n_i - m_i}{1-\sigma} \times \frac{d\sigma}{dz} n_b$$
$$\frac{dll}{dw_0} = \frac{dll}{df}\frac{df}{dz}\frac{\partial z}{\partial w_1} = \frac{\partial z}{\partial w_0} = \frac{m_i}{\sigma} - \frac{n_i - m_i}{1-\sigma} \times \frac{d\sigma}{dz} n_w$$
$$\frac{dll}{dw_0} = \frac{dll}{df}\frac{df}{dz}\frac{\partial z}{\partial b} = \frac{m_i}{\sigma} - \frac{n_i - m_i}{1-\sigma} \times \frac{d\sigma}{dz}$$